



MEADep Application Note:

Using the Weibull Distribution in MEADep

July, 2000

DRAFT

SoHaR Incorporated
8421 Wilshire Blvd., Suite 201
Beverly Hills, CA 90211
323-653-4717

www.sohar.com/meadep
info@sohar.com

Table of Contents

Introduction	1
Model Description.....	1
Results	2
Results with Baseline Parameters	2
Parametric Analysis.....	2
Validation	4
Validation Against Microsoft Excel.....	4
Validation Against RAPTOR 4.....	4
Appendix: RAPTOR and MEADEP Model Results	6

List of Figures

Figure 1. Upper level MEADEP model (3 out of 6)	1
Figure 2. Lower Level MEADEP model (lower level element failure distributions are Weibull distributed)	1
Figure 3. Baseline reliability results.....	2
Figure 4. MEADEP prediction Impact of change of shape factor on 3 out of 6 system reliability for a mission time of 100,000 hours (no replacement)	3
Figure 5. Impact of Redundancy on Subsystem Reliability (mission time of 100,000 hours)	3

List of Tables

Table 1. MEADEP Weibull Results vs. Excel.....	4
Table 2. MEADEP vs. RAPTOR 4 Results.....	4

Introduction

The application note describes the use of the Weibull distribution in creating MEADEP models. This distribution is used where a non-constant (typically increasing) failure rate has been observed. The following section describes an example model that demonstrates use of the Weibull distribution. The subsequent section discusses the baseline results and parametric analyses. The section also summarizes the results of validation calculations against Microsoft Excel and the RAPTOR 4 reliability simulator.

Model Description

To describe the use of the Weibull distribution, we shall use an example system consisting of a satellite 6-axis stabilization sensor system. Only 3 axes are necessary for control through the duration of the mission; hence, the system has 3 out of 6 redundancy.

In MEADEP, this system is represented in a two-level model. The first level is used to represent the 3 out of 6 redundancy as shown in Figure 1. Figure 2 represents the reliability of the lower level blocks using the Weibull distribution.

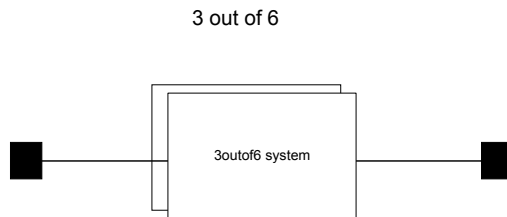


Figure 1. Upper level MEADEP model (3 out of 6)

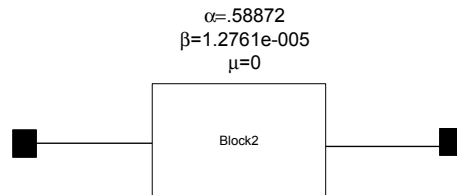


Figure 2. Lower Level MEADEP model (lower level element failure distributions are Weibull distributed)

The Weibull distribution is represented in many forms. MEADEP uses the following representation:

$$R(t) = \exp(-\beta t^\alpha) \quad \text{Eq. 1}$$

The parameter α is called the *shape* factor and the parameter β is called the *scale* factor. Many reliability works use the reciprocals of these quantities. Thus, the parameters in the figure 2 correspond to a shape factor of 1.6986 and a scale factor of 78640 hours in this alternate usage.

It should be noted that we will be creating a *reliability* model for this system because the device is on a satellite and is therefore not economically accessible for repair. Thus, the recovery rate, μ , is set to 0, and when the model is generated in MG, Option 2, a non-repairable model, is chosen.

Results

Results with Baseline Parameters

Figure 3 shows the reliability over time for a 100,000 hour mission for the baseline parameters shown in Figure 2.

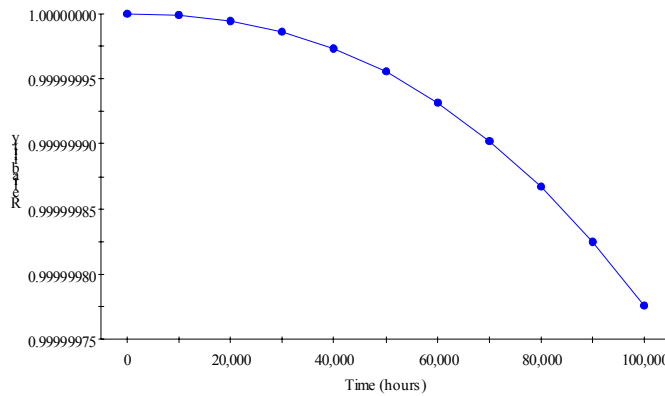


Figure 3. Baseline reliability results

This figure shows a reliability of 0.999997761 at the end of the mission. This high reliability, however, is reflective only of the Attitude Control System gyroscopes. A complete analysis would require consideration of the other elements including the electronics, processor, software, and other components.

Parametric Analysis

Figure 4 shows the results of a calculation of reliability at 100,000 hours using MEADep for the 3 out of 6 system using the scale factor shown in Figure 2 with various shape factors. The figure demonstrates the impact of the shape factor, as well as the capabilities of MEADep. Because MEADep is an analytically oriented tool, the results were calculated instantly.

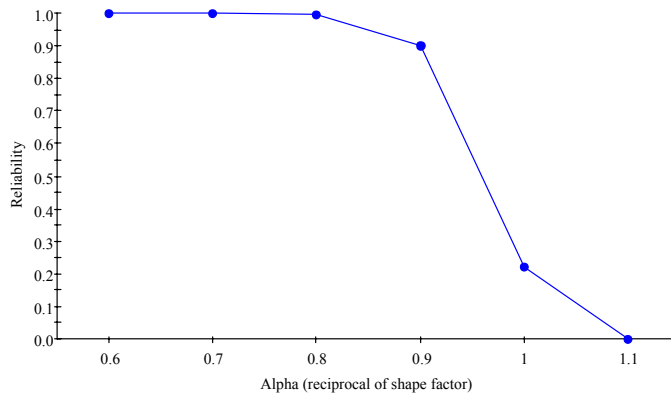


Figure 4. MEADep prediction Impact of change of shape factor on 3 out of 6 system reliability for a mission time of 100,000 hours (no replacement)

Another parametric study that could be performed is the extent of redundancy as shown in Figure 5. With 5 sensors, the reliability is 0.9999865406 whereas for 6 sensors, it is 0.9999997761. It is possible that the 5-sensor reliability would be sufficient in which case the and power resources allocated to the 6th axis could be devoted to another system on the space vehicle.

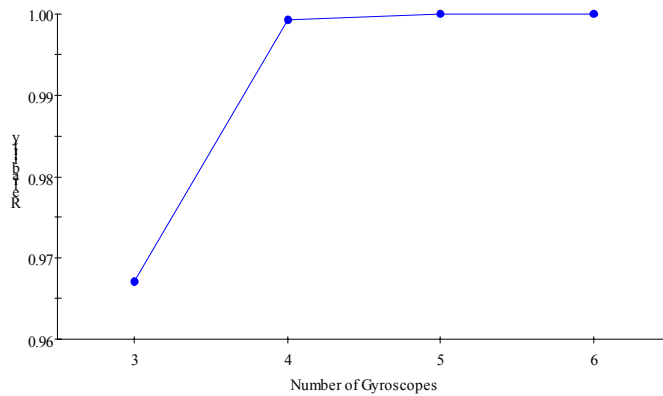


Figure 5. Impact of Redundancy on Subsystem Reliability (mission time of 100,000 hours)

Validation

MEADEP undergoes extensive internal test and evaluation and is also evaluated by many of our customers prior to their initial purchase. This section describes two simple, low cost validation procedures that can be reproduced by the reader to validate MEADEP Weibull results.

Validation Against Microsoft Excel

MEADEP was compared against Microsoft Excel for numerical accuracy for a single Weibull Block diagram. Table 1 shows the results from MEADEP and Excel for the case of a single element with a scale factor corresponding to 78640 hours and shape factors varying from 0.6 to 1.6 using Eq. 1.

Table 1. MEADEP Weibull Results vs. Excel

α	β	Excel evaluation of Eq. 1	MEADEP
0.6	1.27E-05	0.987364	0.987364507
0.7	1.27E-05	0.960586	0.960586232
0.8	1.27E-05	0.880591	0.880592767
0.9	1.27E-05	0.6689	0.668903665
1	1.27E-05	0.280378	0.280382651
1.1	1.27E-05	0.017931	0.01793229
1.2	1.27E-05	3E-06	3.00E-06
1.3	1.27E-05	3.44E-18	0
1.4	1.27E-05	5.95E-56	0

as can be seen from these results, the MEADEP calculation matches the Excel calculation to within 8 significant figures.

Validation Against RAPTOR 4

RAPTOR 4 is a reliability simulation tool developed by the U.S. Air Force Office of Scientific Research. As a simulator, the accuracy of the results are limited and can not be readily used for the very high reliability results described above. However, a comparison can be performed against a one out of 2 repairable system with a Shape factor of 1, a scale of 2 ($1/\beta$ as defined by MEADEP), and an exponential repair MTTR=.5. The analytically calculated availability result was 0.96. The following table shows that MEADEP and RAPTOR agree closely for this example

Table 2. MEADEP vs. RAPTOR 4 Results

Parameter	MEADEP Result	RAPTOR Result
Availability	0.96	0.962771698 (A_{op})
Repair Time	.5 (1/Recovery Rate)	0.486818 (MDT)
Downtime	0.04 (Unavailability)	.03772 (Red Time percent)

The Appendix shows more complete results.

Conclusion

This application note has shown the use of MEADep with the Weibull Distribution. MEADep is one of the few analytical tools that can evaluate reliability models using the Weibull distribution. Analytical methods are in general superior to discrete event simulation (also referred to as “Monte Carlo Simulation” or simply “Monte Carlo methods”) because of the accuracy and repeatability. The example demonstrated the accuracy of MEADep calculations, which far exceeds that of simulators. The validation procedures can be performed by any user as a basis of gaining confidence in the numerical methods of MEADep.

Appendix RAPTOR and MEADEP Model Results

RAPTOR

Input

Name	Failure	Value 1	Value 2	Value 3	Value 4	Repairs	Value 1	Value 2	Value 3	Value 4
unnamed	Weibull	1.00000	2.00000	0.00000		Exponential	0.50000	0.00000		
unnamed	Weibull	1.00000	2.00000	0.00000		Exponential	0.50000	0.00000		

Note: Value 1 and Value 2 refer to the shape and scale parameters of the Weibull Distribution as defined by RAPTOR. Value 3 refers to the MTTR as defined by RAPTOR

Output

```

Unnamed RBD
Fri Jun 25 15:10:21 1999          RESULTS

Results for Unnamed RBD Simulation. Results from 1 run(s):

PARAMETER          MEAN
Ao                  0.962771698
MTBDE               6.828168
MDT                 0.264031
MTBM                1.237496
MRT                 0.486818
% Green Time       65.794877
% Yellow Time      30.482293
% Red Time         3.722830
  
```

MEADEP

Input

```

Block1$k1
Block1$n2
muBlock2          2
betaBlock2        .5
alphaBlock2       1
end
  
```

Shape=1, scale =2 (1/beta), exponential repair, MTTR=.5

Output

Model-Name	Failure-Rate (per hour)	Recovery-Rate (per hour)	Availability	Unavailability
* Weibull 1/2 model				
Main	0.083333333333	2	0.96	0.04

Conclusion: RAPTOR and MEADEP agree for hierarchical RBD with Weibull distribution